CURRENT FEEDFORWARD CONTROL

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BACKGROUND

The problem of developing an industrial servo drive with high performance capabilities for accurate positioning is a subject of much importance. On multi-axis industrial machine servos using classical type 1 servo control, it is a requirement that each machine axis have matched position loop gains to maintain accuracy in positioning. Quite often this means that all machine axes servo drives must have their position loop gains K_v adjusted to the poorest performing axis. Consider the basic approach to the design of a positioning servo drive illustrated in figure 1.

This is the classical type 1 servo, which exhibits characteristic errors ε in position that are well known for various inputs θ_i.

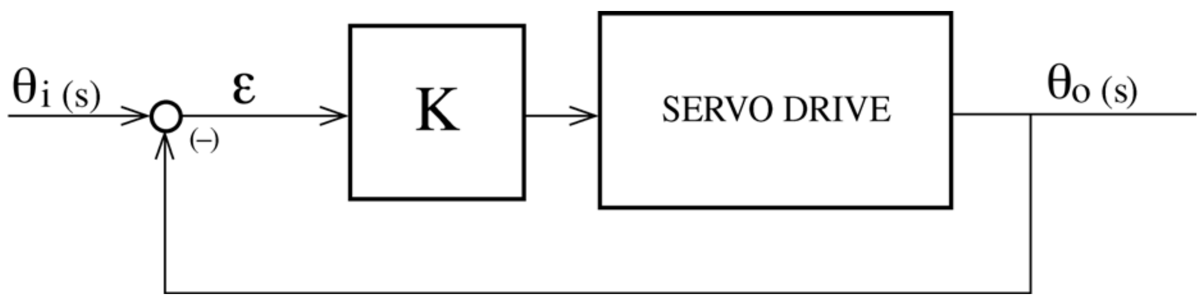

FIGURE 1 TYPE 1SERVO BLOCK DIAGRAM

Consider a simplified block diagram of the system-

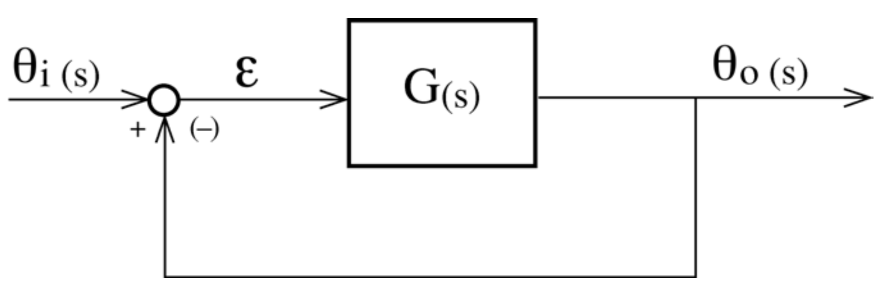

FIGURE 2 SIMPLIFIED POSITION-LOOP BLOCK DIAGRAM

Where $G_{(s)}$ is the servo drive and inner loop transfer function typically of the form-

$$G_{(s)} = \frac{K_v}{sF_{(s)}} \tag{1}$$

$F_{(s)}$ is a polynomial that represents the dynamics of the servo drive and servo plant.

What is really desired of the servo of figure 2 is that $\theta_i = \theta_o$ under all conditions. That is, for any θ_i, ε=0. Clearly, this is not possible for a type 1 servo described by figure 2.

Consider, then, a compensator G_c that could possibly produce a system response as desired, placed cascade with the system as shown in figure 3.

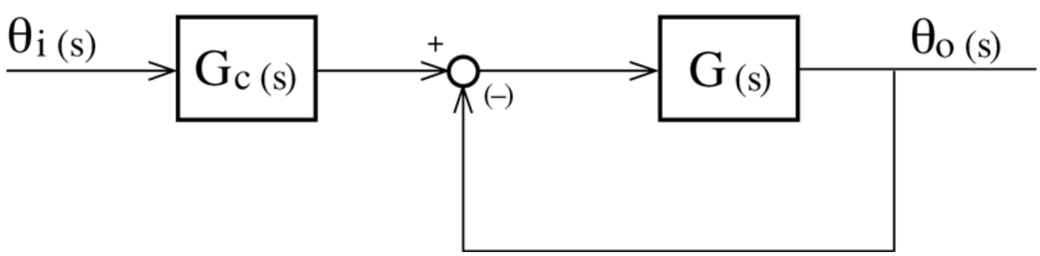

FIGURE 3 COMPENSATOR WITH POSITION SERVO

$$\text{Since } \frac{\theta_{o(s)}}{\theta_{i(s)}} = G_{c(s)} \frac{G_{(s)}}{1+G_{(s)}} = 1 \tag{2}$$

$$\text{Thus: } G_{c(s)} = \frac{1+G_{(s)}}{G_{(s)}} = \frac{1}{G_{(s)}} + 1 \tag{3}$$

Thus, the desired system may be represented in block diagram form as figure 4.

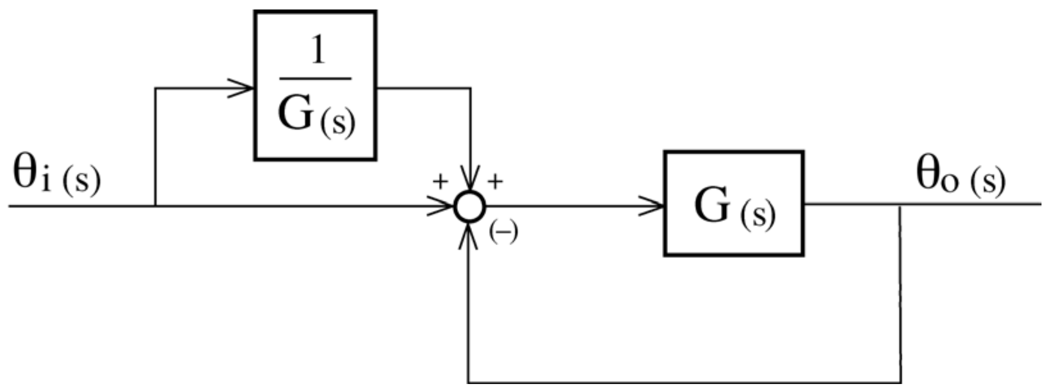

Figure 4 BLOCK DIAGRAM WITH FEEDFORWARD

It is convenient to rearrange the diagram of figure 4 so that the actual error

term $\varepsilon = \theta_i - \theta_o$ appears, as figure 5

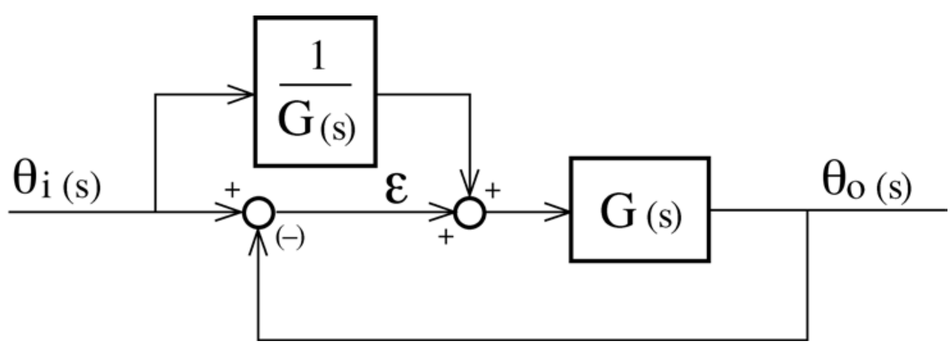

FIGURE 5 ZERO-ERROR FEEDFORWARD BLOCK DIAGRAM

CURRENT FEEDFORWARD

The preceding background for velocity feedforward can be used as a starting point in describing current or acceleration feedforward. In solving the current loop, the forward loop, open loop, and feedback loop must be identified as follows:

The forward servo loop-
The block diagram of figure 6 represents dc and brushless dc motors. All commercial industrial servo drives make use of a current loop for torque regulation requirements.

Figure 6 includes the current loop for the servo drive with PI compensation. Since the block diagram of figure 6 is not solvable, block diagram algebra separates the servo loops to an inner and outer servo loop of figure 7.

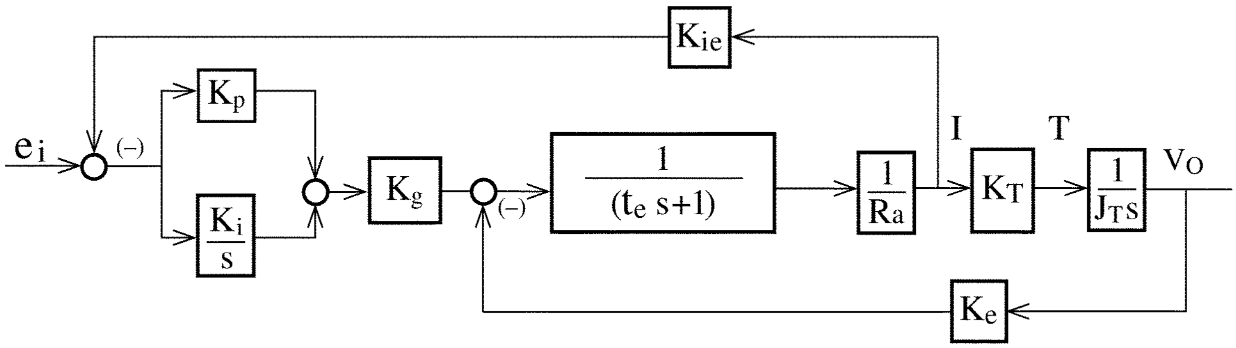

FIGURE 6 MOTOR AND CURRENT LOOP

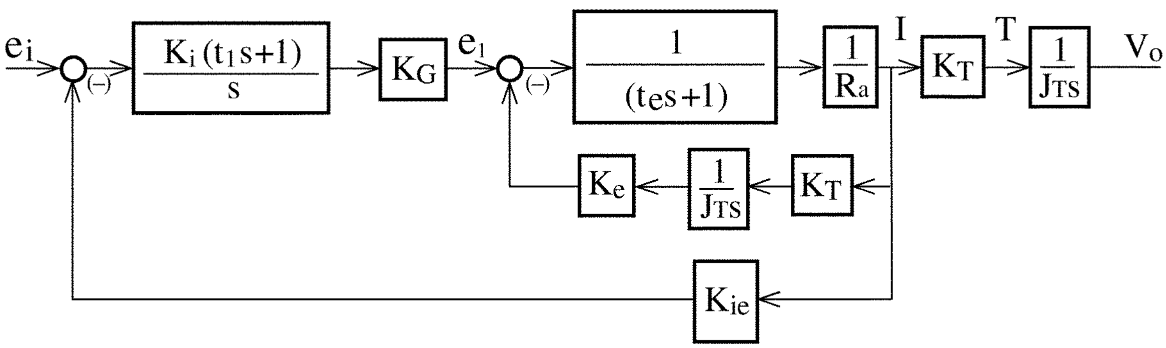

FIGURE 7 MOTOR AND CURRENT LOOP

For this discussion a worst case condition for a large industrial servo axis will be used. The following parameters are assumed from this industrial machine servo application:

Motor - Kollmorgen motor - M607B
Machine slide weight - 50,000 lbs
Ball screw: Length - 70 inches
Diameter - 3 inches
Lead - 0.375 inches/revolution
Pulley ratio - 3.333
J_T = Total inertia at the motor= 0.3511 lb-in-sec^2
t_e = Electrical time constant = 0.02 second = 50 rad/sec
$t_1 = t_e$
K_e = Motor voltage constant = 0.646 volt-sec/radian
K_T = Motor torque constant = 9.9 lb-in/amp
K_G = Amplifier gain = 20 volts/volt
K_{ie} = Current loop feedback constant = 3 volts/40A= 0.075 volt/amp
R_a = Motor armature circuit resistance = 0.189 ohm

K_i = Integral current gain = 735 amp/sec/radian/sec

The first step in the analysis is to solve the inner loop of figure 7. The closed loop response $I/e_1 = G/1+GH$ where:

$G = 1/R_a\,(t_e S+1) = 5.29/[t_e S+1]$ (5.29 = 14.4 dB)

$GH = 0.646 \times 9.9/[0.189 \times 0.3511[(t_e S+1)S]$

$GH = 96/S[t_e S+1]$ 96 = 39dB

$1/H = J_T S/K_e K_T = 0.3511S/0.646 \times 9.9$

$1/H = 0.054\ S$ (0.054 = -25 dB)

Using the rules of Bode, the resulting closed loop Bode plot for I/e_1 is shown in figure 8. Solving the closed loop mathematically :

$$\frac{I}{e_1} = \frac{G}{1+GH} = \frac{1}{R_a(t_e S+1) + K_e K_T / J_T S} = \frac{J_T S}{J_T R_a S(t_e S+1) + K_e K_T}$$

$$\frac{I}{e_1} = \frac{J_T S}{J_T R_a t_e S^2 + J_T R_a S + K_e K_T} = \frac{J_T / K_e K_T\ S}{[(J_T R_a/K_e K_T) t_e S^2 + (J_T R_a/K_e K_T)\,S + 1]}$$

$$\frac{I}{e_1} = \frac{(.3511/.646 \times 9.9)\,S}{t_m t_e\,S^2 + t_m\,S + 1} = \frac{0.054\ S}{0.01 \times 0.02\ S^2 + 0.01\ S + 1}$$

where: $t_m = \frac{J_T R_a}{K_e K_T} = \frac{0.3511 \times 0.189}{0.646 \times 9.9} = 0.01$ sec, $w_m = 1/t_m = 100$ ra/sec

$t_e = 0.02$ sec $w_e = 1/t_e = 50$ rad/sec

For a general quadratic-

$$\frac{S^2}{w_r^2} + \frac{2\ \text{delta}}{w_r} S + 1$$

$w_r = [w_m\ w_e]^{1/2} = [100 \times 50]^{1/2} = 70$ rad/sec

$$\frac{I}{e_1} = \frac{0.054S}{S^2/70^2 + (2delta/70)S + 1}$$

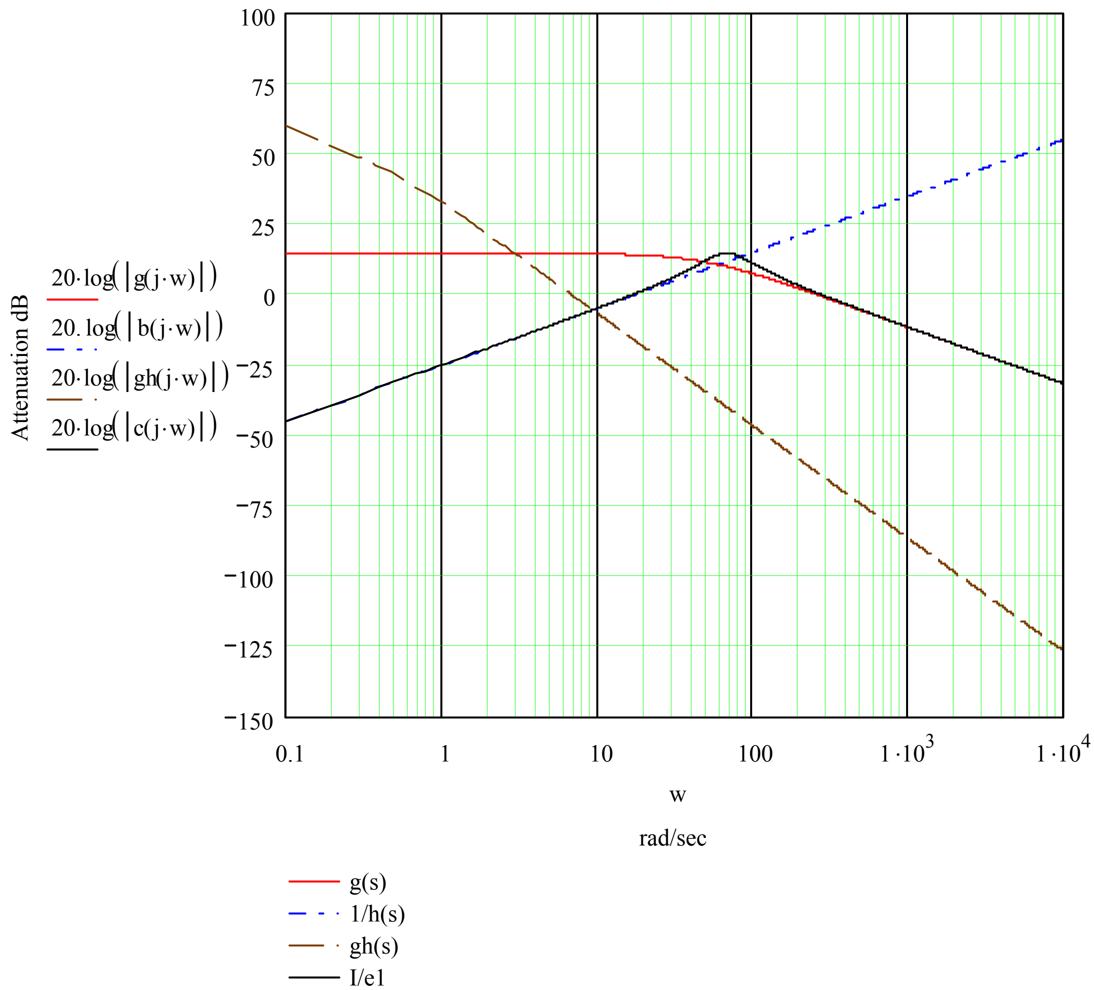

Fig 8 Current inner loop

Having solved the inner servo loop it is now required to solve the outer current loop. The inner servo loop is shown as part of the current loop in figure 9.

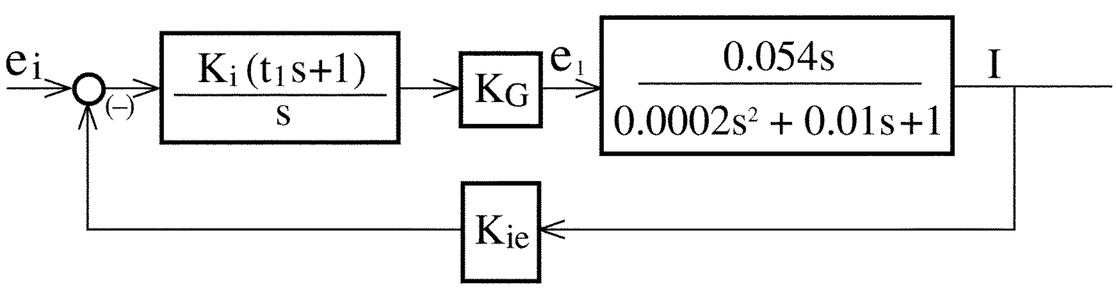

FIGURE 9 CURRENT LOOP

$$G = \frac{K_i\,K_G \text{ x } 0.054\,(0.02S+1)}{0.0002\,S^2 + 0.01\,S + 1} = \frac{735 \text{ x } 20 \text{ x } 0.054\,(0.02S+1)}{0.0002S^2 + 0.01\,S + 1}$$

$$G = \frac{794(0.02\,S+1)}{0.0002S^2 + 0.01S + 1} = \frac{15.88S + 794}{0.0002S^2 + 0.01S + 1}$$

Where: K_G =20 volt/volt
K_{ie} = 3/40 = 0.075 volt/amp
K_iK_G x 0.054 794 (58 dB)
K_i = 794/(20 x 0.054)=735

$$G = \frac{79{,}400S + 3{,}970{,}000}{S^2 + 50\,S + 5000}$$

The open loop-

$$GH = 0.075 \text{ x } \frac{79{,}400S + 3{,}970{,}000}{S^2 + 50S + 5000}$$

$$GH = \frac{5955S + 297{,}750}{S^2 + 50S + 5000}$$

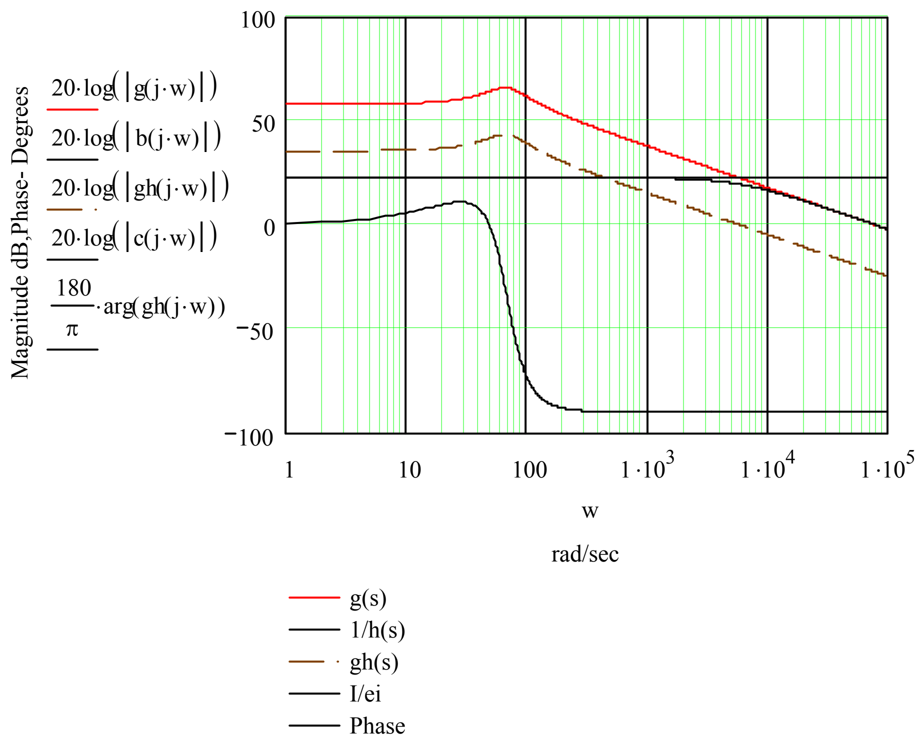

Fig 10 Current loop response

The feedback current scaling is-

$H = 3$ volts/40 amps = 0.075 volts/amp $\quad 1/H = 13.33 = 22.4$ dB

The Bode plot frequency response is shown in figure 10. The current loop bandwidth is 6000 radians/second or about 1000 Hz, which is realistic for commercial industrial servo drives.

The current loop as shown in figure10 can now be included in the motor servo loop with reference to figure 7 and reduces to the motor servo loop block diagram of figure 11.

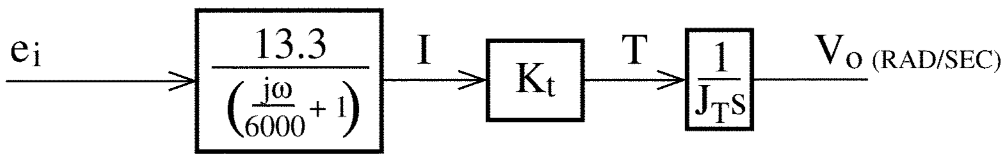

FIGURE 11

The completed motor servo loop has a forward loop only (as shown in figure 11 where:

J_T = Total inertia at the motor = 0.3511 lb-in-sec^2
K_T = Motor torque constant = 9.9 lb-in/amp

$$G = \frac{13.3 \times 9.9}{.3511S\,((jw/6000)+1)} = \frac{375}{S(0.000166S+1)} \quad (51.5\text{ dB})$$

$$G = \frac{375}{0.000166S^2 + S + 0} = \frac{2{,}250{,}090}{S^2 + 6000S + 0}$$

$$\frac{v_o}{e_i} = \frac{K_T}{J_T\,S} \times \frac{|I|}{|v_i|} = \frac{9.9}{0.3511S} \times \frac{13.1(0.02S+1)}{0.00000331S^2 + 0.02S + 1}$$

$$\frac{v_o}{e_i} = \frac{375\,(0.02S+1)}{S\;0.00000331\,(S+50)(S+5991)}$$

$$\frac{v_0}{e_i} = \frac{375\,(0.02S+1)}{S\;0.00000331 \times 50 \times 5991\,((S/50)+1)(((s/5991)+1)}$$

$$\frac{v_o}{e_i} = \frac{375\,(0.02S+1)}{S\,(0.02S+1)(0.000166S+1)}$$

$$\frac{v_o}{e_i} = \frac{375}{S\,((jw/5991)+1)}$$

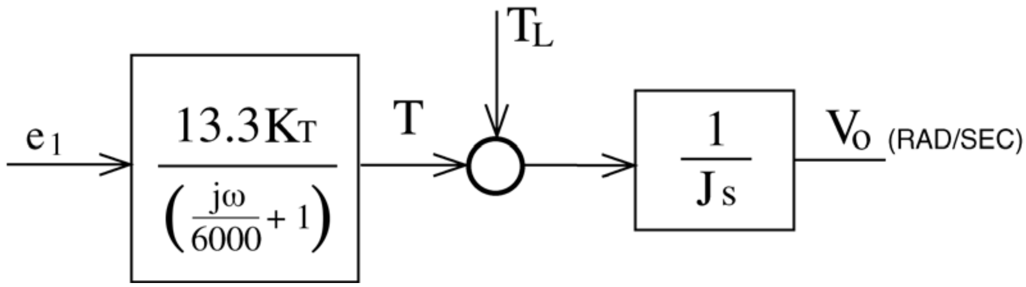

FIGURE 12 SIMPLIFIED CURRENT LOOP

For all practical purposes the current loop response is at 6000 rad/sec, which is much greater that any other response in the servo drive and the current loop could be reduced to just a scale factor 13.3 * Kt.

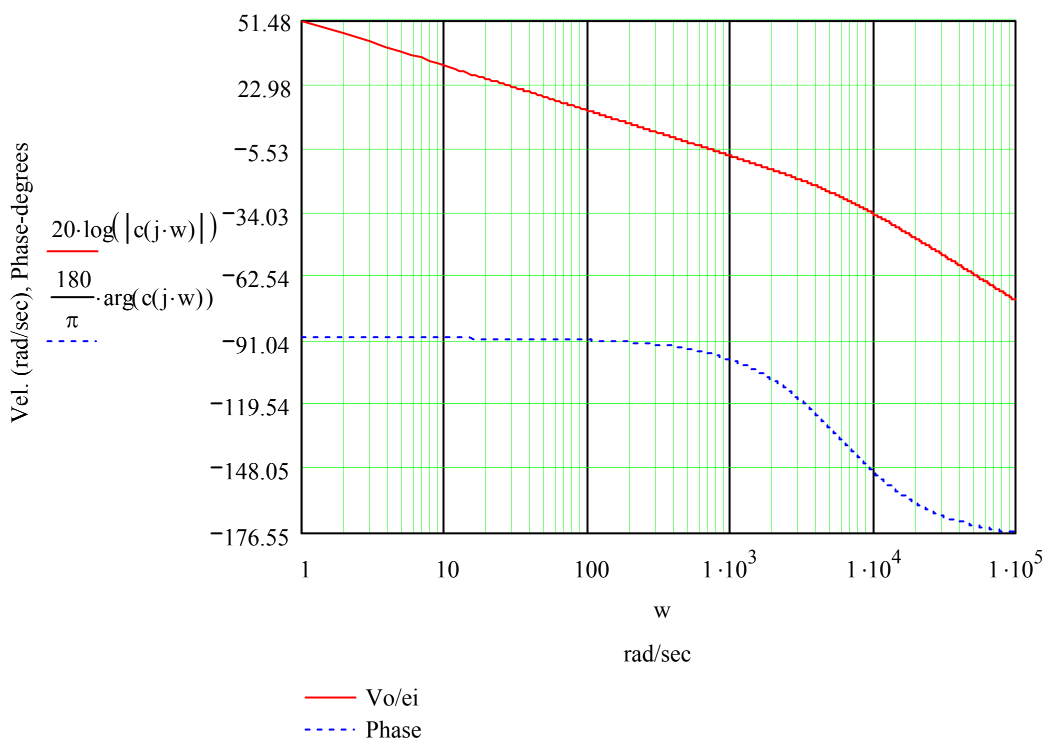

Fig 13 Mot.&I loop freq. response

The Bode frequency response for the motor and current loop is shown in figure 13. The motor and current closed loop frequency response, indicate that the response is an integration which includes the 6000 rad/sec bandwidth of the current loop. This is a realistic bandwidth for commercial industrial servo drives.

For the purposes of this discussion it will be assumed that the motor and current loop with current feedforward are enclosed in a velocity servo loop . Such an arrangement is shown in figure 14. K2 represents the PI compensation.

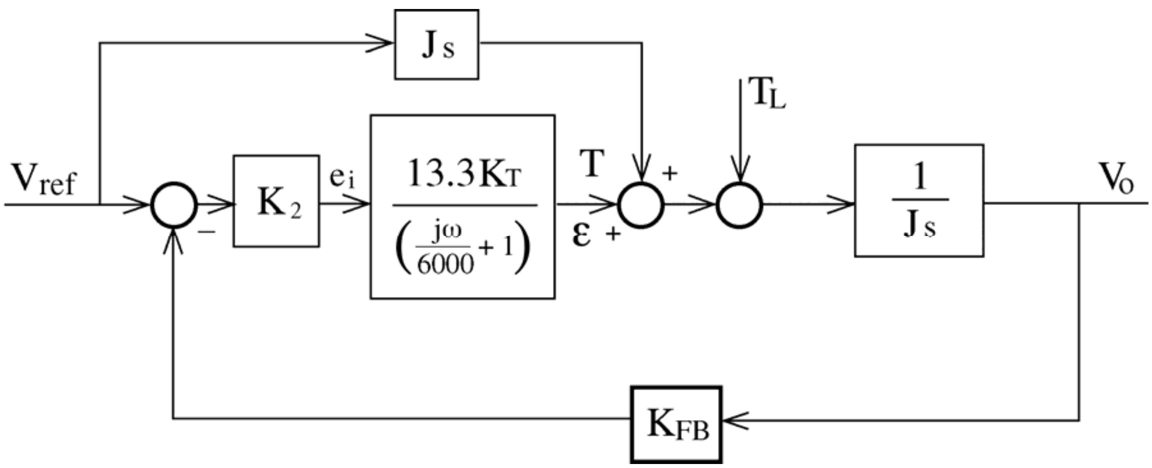

FIGURE 14 CURRENT LOOP WITH CURRENT FEEDFORWARD IMBEDDED IN A VELOCITY LOOP

Using the constants for this particular problem given earlier, figure 14 becomes figure 15.

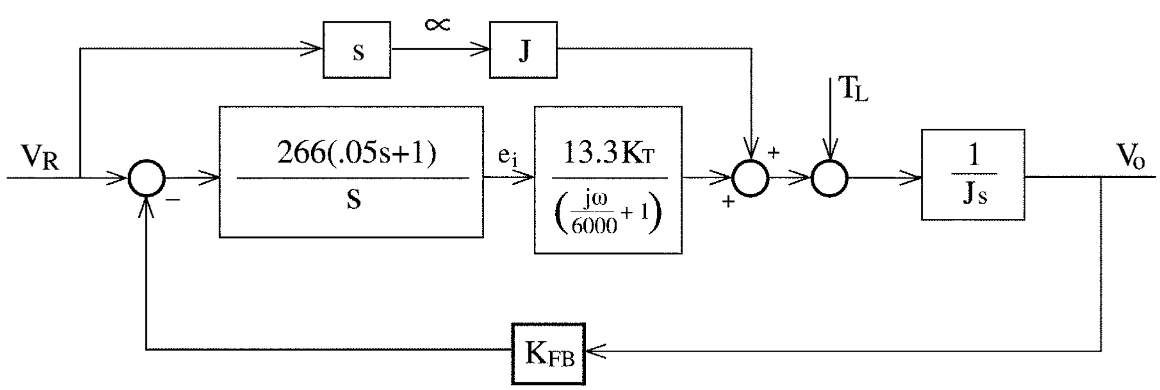

FIGURE 15 VELOCITY SERVO WITH CURRENT FEEDFORWARD AND PI COMPENSATION

Vr= Velocity reference =volts
Vo= Velocity output = rad/sec
J= 0.3511 lb-in-sec^2

Kt= 9.9 lb-in/amp
Kfb=0.0286 v/rad/sec

$$PI = compensation = \frac{266(0.05s+1)}{s}$$

The calculations for figure 15 are as follows:

$$V_o = \left\{ \left[\left(V_r - K_{fb} V_o \right) \frac{266(.05s+1)}{s} \right] * 13.3 K_T + s V_r j + T_L \right\} \frac{1}{js}$$

$13.3K_T = 13.3 * 9.9 = 132$ $\quad$ $266 * 132 = 35024$

$K_{fb} * 266 = .0286 * 35024 = 1001$

$$V_o = \left[V_r 266 \frac{(.05s+1)}{js^2} - K_{fb} V_o 266 \frac{(.05s+1)}{js^2} \right] * 13.3 K_T + \frac{s V_r j}{js} + \frac{T_L}{js}$$

$$V_o = \left[V_r 35024 \frac{(.05s+1)}{js^2} - 1000 V_o \frac{(.05s+1)}{js^2} \right] + \frac{s V_r j}{js} + \frac{T_L}{js}$$

$$V_o \left[1 + \frac{1000(.05s+1)}{js^2} \right] = \frac{V_r 35024(.05s+1)}{js^2} + \frac{s V_r j}{js} + \frac{T_L}{js}$$

$Let\ \ T_L = 0$

$$V_o \left[\frac{js^2 + 1000(.05s+1)}{js^2} \right] = \frac{V_r 35024(.05s+1)}{js^2} + \frac{V_r s^2 j}{js^2}$$

$$V_o = \frac{V_r 35024(.05s+1) + V_r js^2}{js^2 + 1000(.05s+1)}$$

$$V_o = V_r \frac{35024(.05s+1) + js^2}{js^2 + 1000(.05s+1)} = V_r \frac{35024(.05s+1) + js^2}{js^2 + 50s + 1000}$$

$j = 0.3511$

$$\frac{V_o}{V_r} = \frac{35024(.05s + 1) + .35s^2}{.35s^2 + 50s + 1000}$$

$$\frac{V_o}{V_r} = \frac{1751s + 35024 + .35s^2}{.35s^2 + 50s + 1000}$$

$$\frac{V_o}{V_r} = \frac{.35s^2 + 1751s + 35024}{.35s^2 + 50s + 1000}$$

$$\frac{V_o}{V_r s \to 0} = \frac{35024}{1000} = 35.024 = 32dB$$

The velocity servo frequency response bandwidth is about 180 rad/sec or 30 Hz shown in figure 16.

$w := .1, .2 .. 1000($

Scale factor

$k := 35$

$$c(s) := \frac{.30 \cdot k \cdot s^2 + 1751 \cdot s + 35024}{.35 \cdot s^2 + 50 \cdot s + 1000}$$

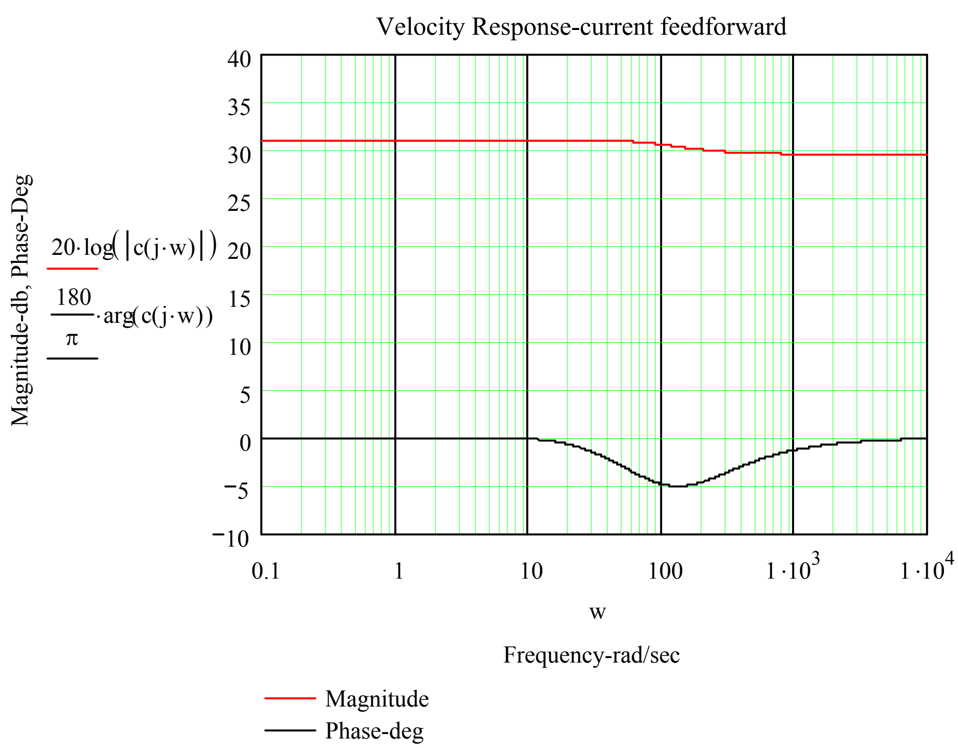

FIGURE 16 VELOCITY SERVO RESPONSE WITH

CURRENT FEEDFORWARD

It should be noted that if the numerator coefficients are divided by the scale factor 35, the transfer function will be equal to 1, accounting for a flat response in figure 16.

The transient velocity servo current feedforward response is shown in figure 17:

$t := 0, .001 .. .1$

$k := 35$

$$c(s) := \frac{.35k \cdot s^2 + 1751 \cdot s + 35024}{.35s^2 + 50 \cdot s + 1000}$$

$$c(t) := \frac{c(s)}{s} \text{ invlaplace}, s \rightarrow 35.024000000000000000 - .24000000000000000000e{-}e^{-71.428571428571428571 \cdot t} \cdot \cosh(47.3803541$$

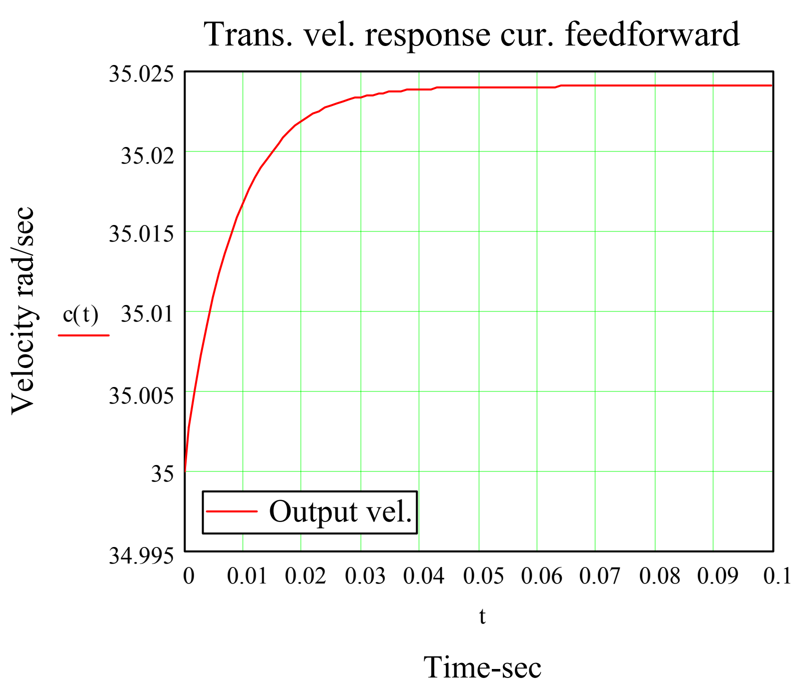

FIGURE 17 TRANSIENT VELOCITY SERVO RESPONSE WITH CURRENT FEEDFORWARD

A test for constant acceleration with the velocity servo is shown in figures 18. This figure indicates that there is excellent tracking between the acceleration command and the actual output velocity, indicating zero error.

$t := 0, .001 .. .1$

$k := 35.024$

$$g(s) := \frac{.35k \cdot s^2 + 1751s + 35024}{.35s^2 + 50s + 1000}$$

$$c(t) := \frac{g(s)}{s^2} \text{ invlaplace}, s \rightarrow 35.024000000000000000t - .20000000000000000000e\text{-}3 + .20000000000000000000e\text{-}3 \, e^{-71.42857142}$$

$$c1(t) := \frac{35}{s^2} \text{ invlaplace}, s \rightarrow 35 \cdot t$$

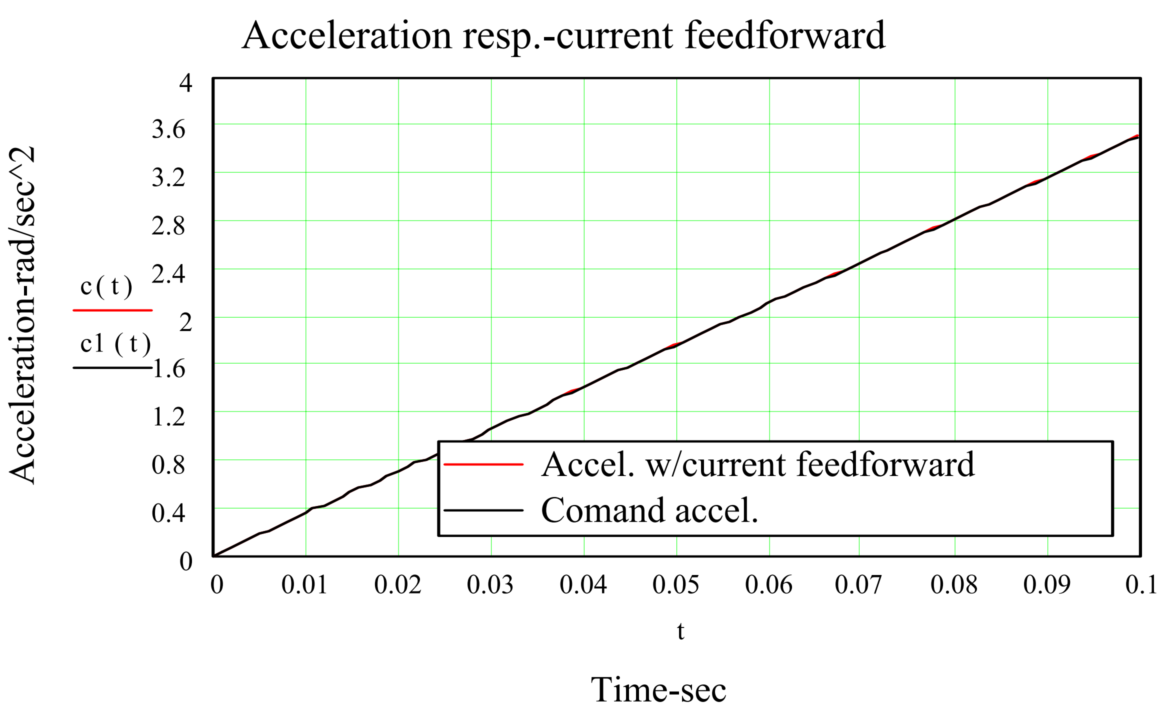

FIGURE 18 VELOCITY SERVO CONSTANT ACCELERATION RESPONSE WITH CURRENT FEEDFORWARD

Further analysis of current feedforward using a basic model instead of an actual real-world model is made with figure 19.

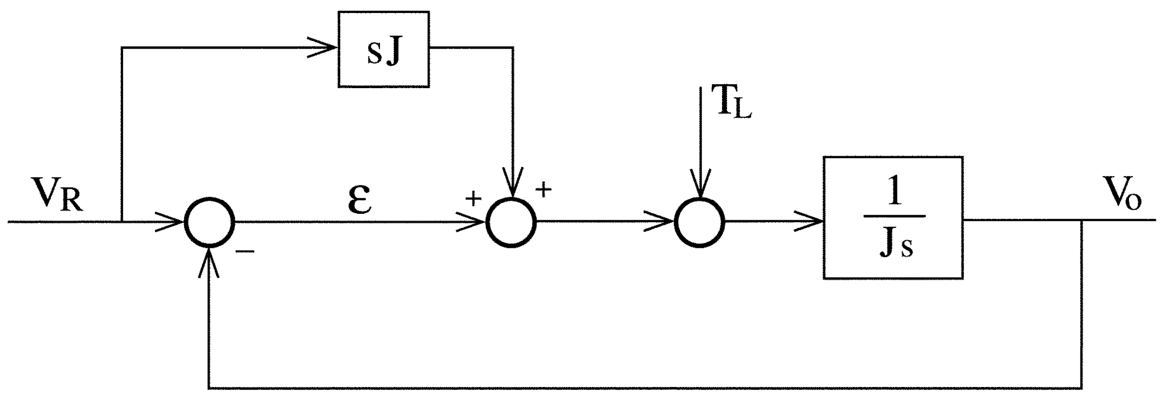

FIGURE 19 BASIC BLOCK DIAGRAM FOR CURRENT FEEDFORWARD

$$[(V_r - V_o) + V_r sj]\frac{1}{sj} = V_o$$

$$\frac{V_r}{sj} - \frac{V_o}{sj} + \frac{V_r sj}{sj} = V_o$$

$$V_r - V_o + V_r sj = V_o sj$$

$$V_r(1 + sj) = V_o(1 + sj)$$

$$V_o(1 + sj) = V_r(1 + sj)$$

$$\frac{V_o}{V_r} = \frac{(1 + sj)}{(1 + sj)} \qquad \therefore V_o = V_r \qquad andvelocityerror = 0$$

Using the forgoing analysis for the velocity servo, a position loop will be added to this real-world servo system. The control block diagram is shown in figure 20.

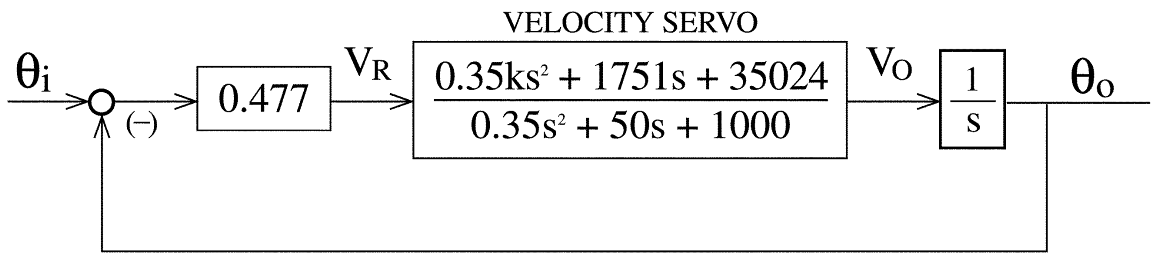

FIGURE 20 POSITION LOOP BLOCK DIGRAM WITH CURRENT FEEDFORWARD IN THE VELOCITY SERVO

The velocity constant for this position loop is: Kv =0.477 * 35 = 16.66/sec = 1 ipm/mil

The forward loop transfer function $g_{s(s)} = \frac{0.477}{s} * \frac{.35ks^2 + 1751s + 35024}{0.35s^2 + 50s + 1000}$

The feedback function $h_{(s)} = 1$

A position loop frequency response is shown in figure 21.

$w := .1, .2 .. 10000$

$$g(s) := .477 \cdot \frac{.35k \cdot s^2 + 1751s + 35024}{s \cdot \left(.35s^2 + 50s + 1000\right)}$$

scale factor $k := 35$

$h(s) := 1$

$$c(s) := \frac{g(s)}{1 + g(s) \cdot h(s)}$$

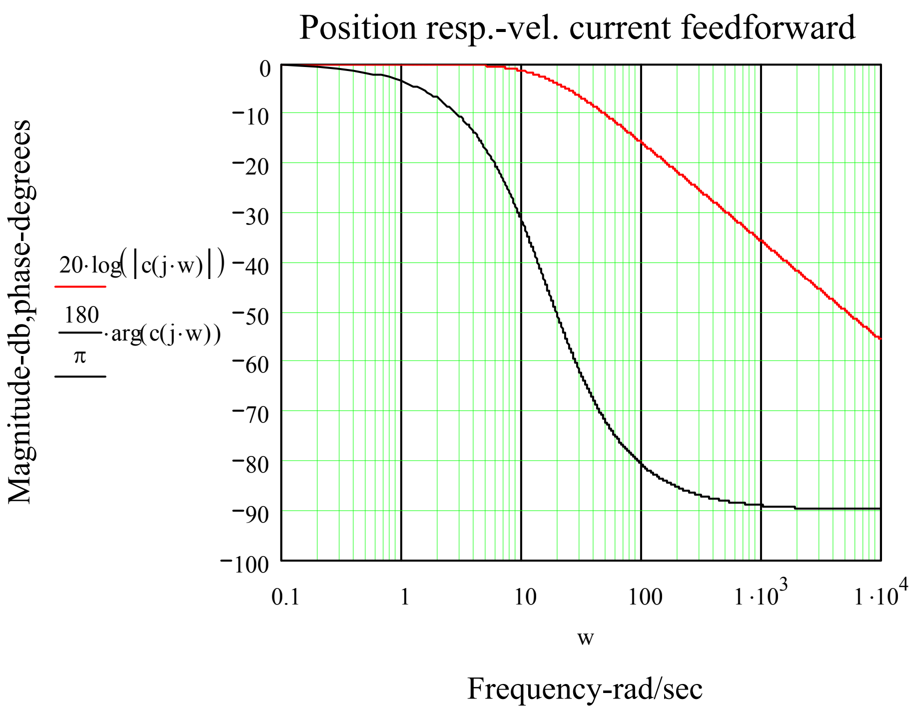

FIGURE 21 POSITION LOOP FREQUENCY RESPONSE

Like wise a position loop transient response is shown in figure 22.

$t := 0, .001 .. .4$

$$g(s) := .477 \cdot \frac{.35k \cdot s^2 + 1751s + 35024}{s \cdot \left(.35s^2 + 50s + 1000\right)}$$

scale factor= $k := 35$

$h(s) := 1$

$$c(s) := \frac{g(s)}{1 + g(s) \cdot h(s)}$$

$$c(t) := \frac{c(s)}{s} \text{ invlaplace}, s \rightarrow 1. + .13081929239008194482e\text{-}3 \, e^{-118.79557060015334717 \cdot t} - .12875217488355099477e\text{-}4 \, e^{-24.0483}$$

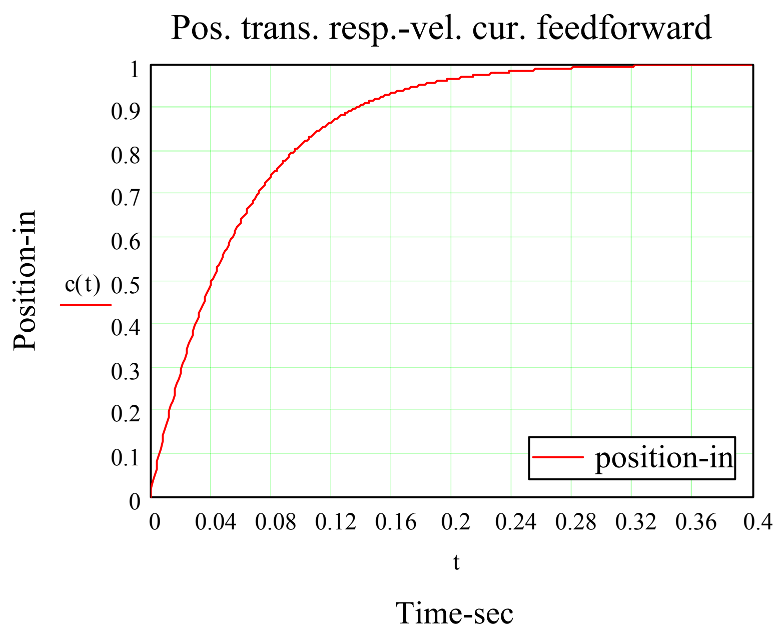

FIGURE 22 TRANSIENT RESPONSE FOR THE POSITION LOOP WITH CURRENT FEEDFORWARD

A position loop response for a constant acceleration is shown in figure 23. It should be noted that the type 1 positioning servo displays the characteristic following error that exist with this type of control.

$t := 0, .001 .. .8$

$$g(s) := .477 \cdot \frac{.35k \cdot s^2 + 1751s + 35024}{s \cdot \left(.35s^2 + 50s + 1000\right)} \qquad \text{scale factor=} \quad k := 35$$

$h(s) := 1$

$$c(s) := \frac{g(s)}{1 + g(s) \cdot h(s)}$$

$$c(t) := \frac{c(s)}{s^2} \text{ invlaplace}, s \rightarrow t - .59857128217799498732e\text{-}1 + .11012135530742850639e\text{-}6 \, e^{-118.79557060015334717050 1 \cdot t} + .535$$

$$c1(t) := \frac{1}{s^2} \text{ invlaplace}, s \rightarrow t$$

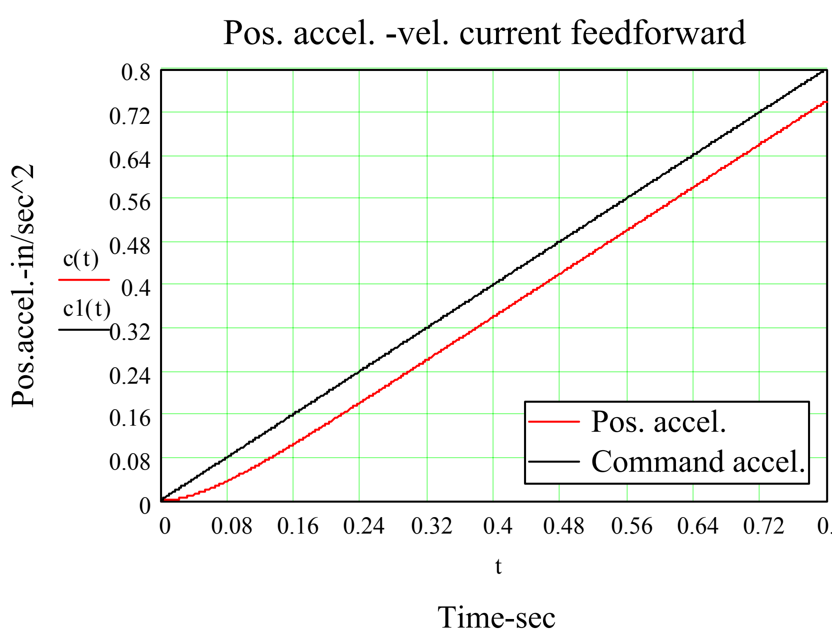

FIGURE 23 POSITION LOOP ACCELERATION RESPONSE WITH CURRENT FEEDFORWARD

Summary

Classic type one positioning systems have a following error. The following error is equal to the velocity/ velocity constant (kv). On a multi axis machine each axis must be tuned with the appropriate velocity constant for a stable servo. To maintain the best possible accuracy for the multi axes operation, all axes servo drives must have the same velocity constant (Kv), which means all axes must be tuned to the poorest performing axis.

The most significant advantage of velocity feedforward control is the ability to minimize or eliminate the following error on multiple axes of a machine allowing for the greatest accuracy attainable. This control concept does not eliminate dynamic problems with the servo plant. Machine resonance problems are another issue.

For many machine requirements velocity feedforward is sufficient to maintain accuracy during constant velocity operation. However, for situations of changing velocity (acceleration/deceleration) such as in a machine contouring operation, acceleration feedforward (also called current or inertia feedforward) can have additional advantages for minimizing servo error. This document illustrates the advantages of both velocity feedforward and acceleration feedforward in a real-world machine example.